Pressure Control Analysis of Cryogenic Storage Systems

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This paper examines self-pressurization of cryogenic storage tanks due to heat leakage through the thermal protection system and the performance of various pressure control technologies intended for application in microgravity environments. Methods of pressure control such as fluid mixing, passive thermodynamic venting, and active thermodynamic venting are analyzed using the homogeneous thermodynamic model. The homogeneous model assumes that the liquid and vapor phases are at a uniform temperature equal to the saturation temperature of the cryogenic fluid at the total tank pressure. Simplified equations suggested in the paper may be used to characterize the performance of various pressure-control systems and to design space experiments for the development of low-gravity cryogenic fluid management technologies.

Nomenclature

C = coefficient defined in Eq. (9)
E = total system energy
h = specific fluid enthalpy
m = mass flow rate
P = tank pressure
Q = tank heating rate
r_1 = parameter defined in Eq. (15)
r_2 = parameter defined in Eq. (17)
t = time
u = specific internal energy of tank fluid
V = tank fluid (liquid and vapor) volume, not including volume occupied by internal tank hardware
v = average fluid (liquid and vapor) specific volume defined in Eq. (A5)
W = power, rate of work done on control volume fluid
x = quality
β = tank fill fraction, percentage fill/100
ρ = average tank fluid (liquid and vapor) density defined in Eq. (A4)
ρ^* = density ratio, $\rho_g/(\rho_f - \rho_g)$
ϕ = energy derivative defined in Eq. (5)

Subscripts

f = saturated liquid
fg = liquid-to-vapor phase change
g = saturated vapor
hsp = homogeneous self-pressurization
i = inlet
max = maximum
mix = mixer, mixing
o = outlet
tvs = thermodynamic vent system
w = wall

Introduction

THE thermal environment of space produces inevitable heat transfer through the thermal protection system of a cryogenic storage tank, which results in continuous tank pressure rise. Tank pressurization that is due solely to heating is known as self-pressurization. The pressure rise rate is governed by the complex interaction of external heat leak, fluid temperature stratification, and interfacial heat and mass transfer. If the required storage duration of a space mission is longer than the period in which the tank pressure reaches its allowable maximum, an appropriate pressure-control method must be applied. Therefore, predictions of the self-pressurization rate and the performance of pressure-control techniques in cryogenic tanks are required for development of long-duration cryogenic fluid storage technology and planning of future space exploration missions.

In this paper, pressure-control analyses are presented for cryogenic storage systems in low gravity, where it is impractical to preferentially vent vapor as a means of controlling tank pressure. Ideally, if a tank is well mixed so that the fluid temperature is isothermal, the homogeneous thermodynamic model may be used for the prediction of pressure change rate. The analyses are based on thermodynamic principles only; therefore factors such as surface heating uniformity, tank shape, wetted tank wall area, and liquid position within the tank are not considered. The tank contains a pure fluid and no noncondensable gas species is present. The total tank heating rate is known. The liquid and vapor phases are considered to be at the same temperature, although separation of the phases is allowable. All fluid properties are averaged based on liquid fill fraction. If the fluid is isothermal, there will be no thermal stratification in either fluid phase. Since the analyses are intended for low-gravity applications, the fluid may not be settled. It is assumed that the tank pressure is not affected by systems (such as gas pressurization systems) other than those discussed herein. It is further assumed that vapor-free liquid can be supplied to the various pressure-control systems by liquid-acquisition devices regardless of tank fill level, liquid–vapor interface location, and liquid distribution.

Pressure control technologies being developed at NASA include high-performance thermal insulation, fluid mixing, and both passive and active thermodynamic vent systems (TVS).[1−3] Schematic representations of self-pressurization and pressure control technologies for application in microgravity environments are illustrated in Fig. 1. Advanced multilayer thermal insulation systems can greatly reduce the heat leak to a cryogenic storage tank. Vapor-cooled shields are sometimes utilized to further intercept incoming thermal energy. For

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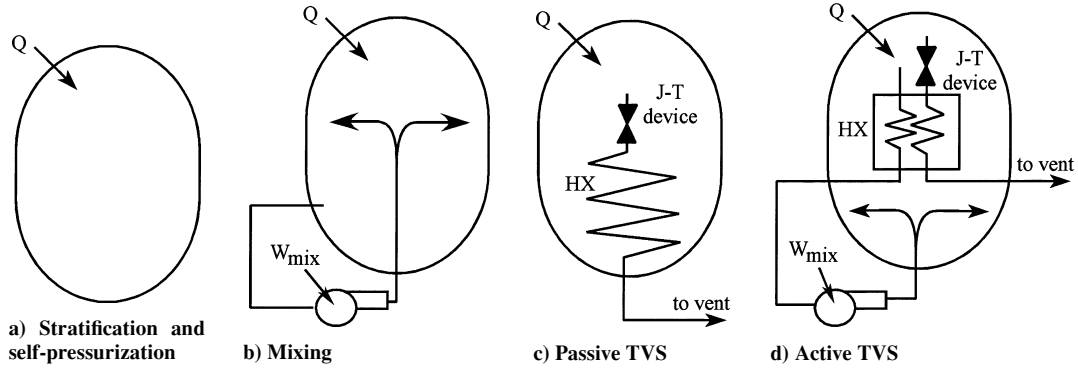‡Aerospace Engineer, Microgravity Fluid Physics Branch, M.S. 77-5.

Fig. 1 Schematic representation of self-pressurization and pressure control technologies (mixer shown outside of tank for clarity).

a given cryogenic storage tank, the heat leak creates thermal stratification in the fluid, which generally results in a pressure rise rate in the tank higher than that for a well-mixed condition (Fig. 1a). Fluid mixing (Fig. 1b) can be used to circulate and mix the tank fluid, destroying fluid temperature stratification. Spacecraft maneuvers, such as attitude and thermal control, may also promote fluid motion and mixing in storage tanks. Mixing induces condensation at the liquid–vapor interface, resulting in reduction of tank pressure. Therefore, interface condensation plays a key role in controlling the rate of change of tank pressure.

Since mixer power is required to circulate the tank fluid, a certain amount of energy is added to the system. This additional energy imparted to the fluid eventually becomes heat and increases the net fluid energy. Thus, fluid mixing can only temporarily reduce tank pressure and is of interest for short-term storage. The design of the mixer influences the efficiency and performance of a fluid-mixing pressure control system.

In the passive TVS concept (Fig. 1c) a small amount of tank liquid is withdrawn using a liquid-acquisition device (continuously or intermittently) and passed through a Joule–Thomson device, resulting in a lower pressure and temperature two-phase fluid. This two-phase mixture then flows through a TVS heat exchanger to cool the tank fluid via passive energy exchange, primarily by thermal conduction or free convection, as shown in Fig. 1c. The fluid mixture is evaporated in the heat exchanger and the resulting vapor is then either vented overboard directly or passed through vapor-cooled shields (not shown in Fig. 1c) to intercept heat leak into the tank and then vented overboard. With this method, energy is removed from the system by sacrificing a small amount of liquid. Venting of vapor is always ensured regardless of liquid position in microgravity. Thus, the passive TVS may be used in a long-term storage system. However, an undesirable weight penalty may occur, because a large heat exchanger surface area is required to vaporize the vented fluid. Also, the relatively slow passive heat transfer processes on the tank side of the heat exchanger may lead to unacceptably long lag times during attempted pressure control.

The TVS and fluid-mixing techniques may be combined to produce an efficient and fast-response pressure-control system. This approach is called the active TVS. In the specific configuration shown in Fig. 1d, the TVS heat exchanger is designed to remove energy from the mixer flow; the resulting subcooled liquid is then circulated back to the tank through a nozzle to mix the tank fluid and induce condensation. Optimal design of the heat exchanger and mixer is quite important for proper active TVS performance.

This report presents analyses of tank self-pressurization and the aforementioned pressure-control methods. Simplified equations based on the homogeneous model are derived to provide a design tool for space-based cryogenic fluid storage systems and for the design of relevant space experiments. Some examples with representative values of the pertinent parameters to characterize the performance of pressure-control systems are also included.

Homogeneous Model

A general form of the model for a cryogenic storage tank can be obtained by applying the first law of thermodynamics and the law of conservation of mass to a control volume that contains the tank liquid and vapor contents, assumed to be in a homogeneous state. Internal tank hardware is not included in the control volume. The control surface corresponds to the inside surface of the tank wall. The first law for a variable volume with one inlet and one outlet (neglecting kinetic and potential energy terms) is

$$\frac{dE}{dt} \cong \rho V \frac{du}{dt} + \rho u \frac{dV}{dt} + V u \frac{d\rho}{dt}$$

$$= Q + W + m_i h_i - m_o h_o - P \frac{dV}{dt} \quad (1)$$

Equation (1) applies to the entire fluid volume, including both liquid and vapor phases. All thermodynamic properties are mass-averaged based on liquid fill fraction. Power input to the control volume (e.g., mixing) is accommodated using a positive value for W.

Conservation of mass for a compression or expansion process is expressed as

$$\frac{dm}{dt} = m_i - m_o = V \frac{d\rho}{dt} + \rho \frac{dV}{dt} \quad (2)$$

If specific internal energy is considered as a function of density and pressure, $u = u(\rho, P)$, then

$$\frac{du}{dt} = \left(\frac{\partial u}{\partial \rho}\right)_P \frac{d\rho}{dt} + \left(\frac{\partial u}{\partial P}\right)_\rho \frac{dP}{dt} \quad (3)$$

The general-purpose homogeneous equation for pressure change rate is obtained by combining the preceding equations and using the relationship $h = u + P/\rho$:

$$\frac{dP}{dt} = \frac{\phi}{V} \left\{ Q + W + m_i \left[h_i - h - \rho \left(\frac{\partial h}{\partial \rho}\right)_P \right] \right.$$

$$\left. - m_o \left[h_o - h - \rho \left(\frac{\partial h}{\partial \rho}\right)_P \right] + \rho^2 \left(\frac{\partial h}{\partial \rho}\right)_P \frac{dV}{dt} \right\} \quad (4)$$

where the energy derivative ϕ is defined as

$$\phi = 1 \bigg/ \rho \left(\frac{\partial u}{\partial P}\right)_\rho \quad (5)$$

Since ρ, and therefore ϕ, are mass-averaged properties for the total tank fluid mass, their values vary with tank fill fraction. A procedure for numerical approximation of the energy derivative is given in the Appendix. The total pressure change is obtained by integrating an appropriate form of Eq. (4) using a known initial condition.

Figure 2 shows the parameter ϕ as a function of average fluid density and tank pressure for hydrogen at saturated, two-phase conditions. It is seen that ϕ decreases with increasing average fluid density ρ and increases with increasing tank pressure P. The energy derivative is discontinuous at both the saturated liquid and saturated vapor points. The curves shown are for a liquid and vapor mixture of pure hydrogen. Because the fluid is a single species, the total tank pressure equals the saturation pressure. The curves should
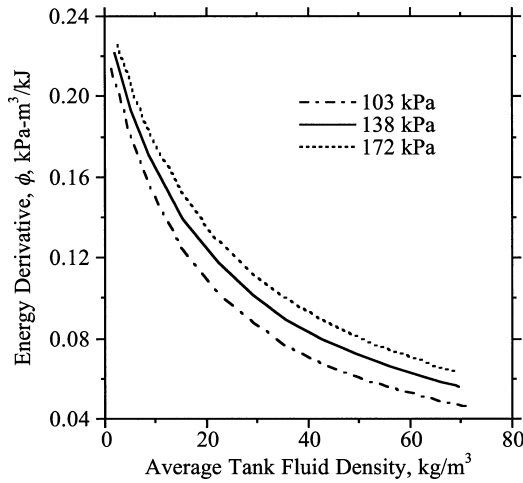
Fig. 2 Functional dependence of energy derivative on density and tank pressure for hydrogen.

Fig. 3 Pressure rise dependence on fill level and heat leak (two times homogeneous pressure rise rate, $V = 52$ m^3).

not be used for a system containing additional species, such as an inert pressurant gas. Energy derivatives for other cryogens such as nitrogen and oxygen exhibit similar behavior.

The homogeneous model is applicable when isothermal conditions exist in the tank. Both liquid and vapor in the tank can be conditioned to a uniform temperature (the saturation temperature, since the tank contains a pure fluid) by sufficient circulation of the tank contents through the use of a mechanical mixer or possibly as a result of fluid motion resulting from spacecraft maneuvering operations. Equation (4) is valid over the entire range of tank fill fractions, but the numerical calculation of ϕ (outlined in the Appendix) becomes impractical as the limiting conditions $\beta \rightarrow 0$ and $\beta \rightarrow 1$ are approached. This is not a significant difficulty, because cryogenic storage tanks are not hard-filled and liquid-acquisition devices cannot completely remove all liquid from a tank.

Self-Pressurization

For self-pressurization of a closed constant-volume cryogenic tank experiencing heat leak only, there is no work done on the fluid by pumping and no mass flow into or out of the tank ($dV/dt = W = m_i = m_o = 0$), and so Eq. (4) reduces to

$$\left(\frac{dP}{dt}\right)_{\text{hsp}} = \frac{\phi Q_w}{V} \tag{6}$$

Given a fixed volumetric heating rate, Q_w/V, the homogeneous model shows that the self-pressurization rate is increasing with time because the energy derivative ϕ increases with increasing tank pressure. Usually, the homogeneous model, which assumes a uniform temperature in the tank, will give the lowest pressure rise rate for a tank. Ground tests with cryogenic hydrogen have shown that the actual pressure rise rate spans a range from approximately 1 (Refs. 4, 5) to more than 10 (Ref. 6) times faster than the homogeneous model prediction for well-insulated tanks. In low-gravity experiments,[7] the pressure rise rate has been found to be lower than that in normal gravity at the same total heat input due to reduced buoyancy and an increase in the liquid-wetted wall area. The homogeneous rate is a convenient baseline, as it represents the slowest pressure rise rate in most normal-gravity situations.

Equation (6) indicates that the pressure rise rate increases linearly with the tank volumetric heating rate Q_w/V and the parameter ϕ. For the purpose of illustration, a self-pressurization rate of two times the homogeneous prediction is assumed as a characteristic rate for the examples provided herein. Figure 3 shows the pressure rise of a spherical 52-m^3 liquid hydrogen tank (a typical size for future spacecraft), initially at 101 kPa, at various fill levels (by volume) and heating rates. The range of heating rates corresponds to the expected heat flux range[8] in full-scale applications. It is seen that lower liquid fill level (yielding smaller ρ and larger ϕ) and higher heating rate (yielding greater Q_w/V) result in higher tank pressure
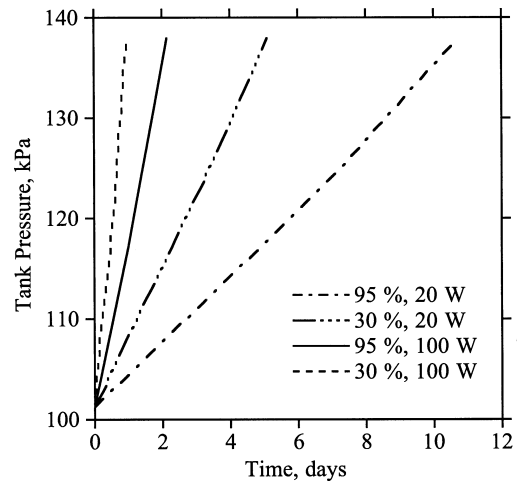
rise rate. For 95% fill level, if the wall heating rate is reduced from 100 to 20 W, the mission duration can be increased from 2.1 to 10.7 days, assuming that the maximum allowable tank pressure is 138 kPa. This implies that a well-designed thermal protection system can reduce the heat leak and provide a significant extension of the storage duration. This benefit occurs at all fill levels and is enhanced if the allowable maximum tank pressure rise is larger.

Fluid Mixing

Mixing, such as that induced by an axial jet, may be used to extend the storage duration. As the tank fluid is mixed, thermal stratification is reduced and interfacial condensation is promoted. It is assumed that the entire tank can be completely mixed so that a homogeneous state can be achieved. When the fluid temperature becomes uniform, condensation ceases and continued operation of the mixer will not produce a further decrease in tank pressure.

Continuous and intermittent mixing are two possible modes of operation for fluid mixing. In the continuous mixing mode, a well-mixed homogeneous state is maintained. Intermittent mixing is a cyclic process in which fluid stratification occurs until a specified operating pressure is reached, followed by a mixing period that lasts until a homogeneous state is achieved. The cycle repeats until the pressure in the homogenous state approaches the maximum allowable pressure. In either mode, operation of the mixer results in heat input from dissipation of the mixer power, which augments the pressure rise rate (as compared to the homogeneous self-pressurization rate). Thus, the mixer power W_{mix} (corresponding to the required jet flow rate) and the mixing time t_{mix} are two key variables governing mixing performance. For intermittent mixing, mixing time is defined as the period from when the mixer is turned on to when the tank pressure reaches the value corresponding to a homogeneous state. For an efficient mixing system, the total energy input by the mixer, $W_{\text{mix}}t_{\text{mix}}$, should be kept as low as possible.

Equation (4), when applied to a mixing operation within a fixed-volume tank, reduces to

$$\frac{dP}{dt} = \frac{\phi}{V}(Q_w + W_{\text{mix}}) = \frac{\phi Q_w}{V}\left(1 + \frac{W_{\text{mix}}}{Q_w}\right)$$

$$= \left(\frac{dP}{dt}\right)_{\text{hsp}}\left(1 + \frac{W_{\text{mix}}}{Q_w}\right) \tag{7}$$

It predicts that $dP/dt > (dP/dt)_{\text{hsp}}$. Therefore, if the tank is already in a homogeneous state, fluid mixing will result in an increase in pressure because of the additional heat input from the mixer. If the mixer power-to-heating-rate ratio is large, the pressure rise rate with continuous mixing could be higher than that with stratified self-pressurization. A large mixer power-to-heating rate ratio could occur due to poor mixer design and/or very high-performance
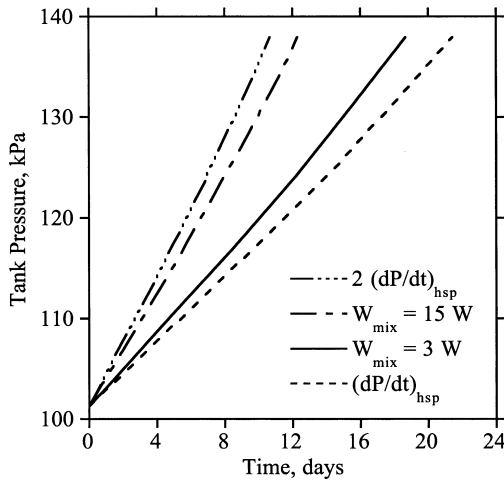
Fig. 4 Self-pressurization and the effect of continuous mixing (95% fill, $Q_w = 20$ W, $V = 52$ m^3).

Fig. 5 Comparison of intermittent and continuous mixing (95% fill, $Q_w = 20$ W, $V = 52$ m^3).

thermal protection systems. Equation (7) assumes that the recirculating fluid does not exit and re-enter the control volume. For an external pump, W_{mix} would be replaced by $m_{mix}\Delta h_{mix}$, where $\Delta h_{mix} = h_{mix,i} - h_{mix,o}$ in the equation. It is possible to analyze various mixing processes combined with self-pressurization periods by the combined use of Eqs. (6) and (7), as is discussed in the following example.

Several possible mixing situations are considered. In each situation the tank is initially 95% full and saturated at 101 kPa. Homogenous self-pressurization from 101 kPa to a specified operating pressure of 138 kPa would require 21.4 days, as shown in Fig. 4, and is considered the maximum possible duration for storage without venting.

1) If the mixer is off, $W_{mix} = 0$, tank pressure rises at an assumed rate of two times $(dP/dt)_{hsp}$ and will reach the operating pressure of 138 kPa in 10.7 days, as shown in Fig. 4.

2) If the mixer runs continuously at $W_{mix} = 15$ W, tank pressure will reach 138 kPa in 12.3 days, as shown in Fig. 4. This is only 1.6 days longer than case 1.

3) If the mixer runs continuously at a lower power, such as $W_{mix} = 3$ W, 18.7 days will elapse before the pressure reaches 138 kPa as shown in Fig. 4. This is 8.0 days longer than in case 1. It has been assumed that homogenous conditions can be attained at this lower mixer power.

4) Based on available ground-based mixing time correlations,[9−11] the mixing time for the system under consideration is less than 2 h for a wide range of jet flow rates. Because the mixing time is quite short, the total energy input by the mixer, $W_{mix}t_{mix}$, is negligibly small compared with the tank internal energy. Thus, the tank pressure is reduced to about 119 kPa (homogeneous condition) after the first mixing cycle, as shown in Fig. 5. If the mixer is run intermittently, only two mixing cycles are required to achieve a storage duration similar to that for case 3.

The choice of intermittent vs continuous mixing is determined by comparing the energy inputs for each method. Considering that the power required to achieve a well-mixed state from a thermally stratified state is not likely to be substantially higher than that needed to maintain homogeneous conditions, the intermittent mode is expected to be more efficient.

In reality, when the mixer is turned on, interface condensation begins and tank pressure starts to decrease. As the fluid is mixed to a certain degree, the pressure reaches a minimum value that is higher than the corresponding value for the homogeneous self-pressurization process. At this moment, the competing effects of wall heat flux (increasing tank pressure) and induced interface condensation (decreasing tank pressure) on the pressure change become balanced. As the mixer continues to operate, the rate of tank pressure change becomes positive, due to diminishing interfacial condensation. Also, the mixer power input generates heat, inducing liquid
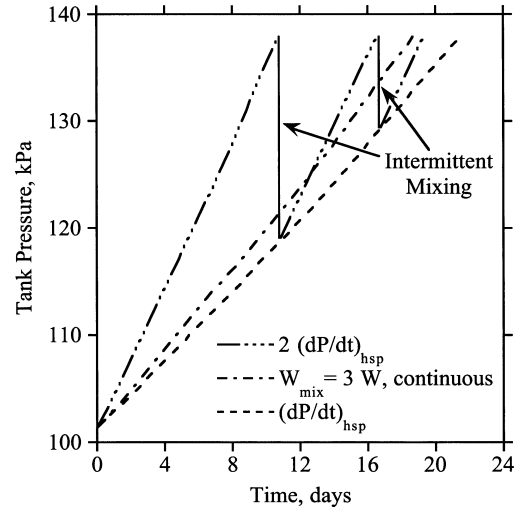
evaporation instead of condensation as homogeneous conditions are neared. Therefore, mixing time should be established as the time period from when the mixer is turned on to when the tank pressure reaches a minimum. Over- or under prediction of the mixing time and/or mixer power will affect mixing performance efficiency.

Passive Thermodynamic Vent System

A constant-volume tank of homogeneous fluid subject to either direct venting or passive TVS operation is governed by the following form of Eq. (4):

$$\frac{dP}{dt} = \frac{\phi}{V}\left\{ Q - m_o\left[h_o - h - \rho\left(\frac{\partial h}{\partial \rho}\right)_P \right] \right\}$$

$$= \frac{\phi}{V}\left[Q_w - Q_{tvs} - m_{tvs}h_{fg}(x + \rho^*) \right] \quad (8)$$

where x is the quality of the fluid entering the Joule-Thompson (J–T) device, $\rho^* = \rho_g/(\rho_f - \rho_g)$, and Q_{tvs} is the heat removal rate of the TVS heat exchanger. A work term is not included since the vent flow is passively driven by the tank-to-vent pressure drop. If vapor-cooled shields are utilized, the vent flow will alter the heat leak rate, Q_w. If $Q_{tvs} = 0$, then Eq. (8) represents the homogeneous state of direct venting with $x = 1$ for saturated vapor only and $x = 0$ for saturated liquid only.

For normal TVS operation, liquid is withdrawn from the tank through a liquid-acquisition device so that the fluid entering the J–T device has the quality $x = 0$. The rate of heat removal by the lower pressure two-phase flow as the fluid passes through the heat exchanger can be expressed as

$$Q_{tvs} = m_{tvs}\Delta h_{tvs} = C m_{tvs}h_{fg} \quad (9)$$

In Eq. (9), C is set equal to $\Delta h_{tvs}/h_{fg}$, where Δh_{tvs} is the change in enthalpy in the TVS heat exchanger and h_{fg} is the latent heat of vaporization corresponding to the tank pressure. The coefficient C is a system parameter and its value depends on the performance of the TVS heat exchanger, the J–T device, and the tank and vent pressures. For a properly designed space-based system, typical values of C may vary from 0.8 to 1.1. Thus, Eq. (8) becomes

$$\frac{dP}{dt} = \frac{\phi}{V}Q_w\left[1 - \frac{m_{tvs}h_{fg}}{Q_w}(C + \rho^*) \right]$$

$$= \left(\frac{dP}{dt}\right)_{hsp}\left[1 - \frac{m_{tvs}h_{fg}}{Q_w}(C + \rho^*) \right] \quad (10)$$

These equations may be used to compare the performance of a passive TVS to that of an ideal TVS for a isothermal fluid system. In
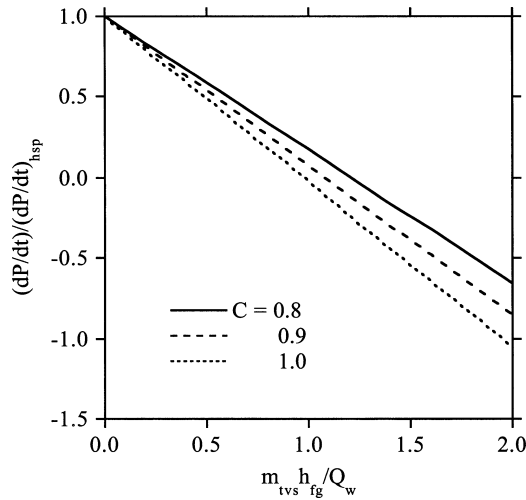
Fig. 6 Passive TVS performance for hydrogen at 138 kPa.

Fig. 7 Effect of the parameter r_1 on pressure change rate.

an ideal TVS, the vapor leaves the heat exchanger at a temperature equal to the tank fluid temperature; that is, Δh_{tvs} is at a maximum. For a cryogenic hydrogen system the maximum value of C (an ideal system) can be approximated by

$$C_{max} \approx 1 + 4.3 \times 10^{-4} \Delta P \qquad \text{(for } P < 400 \text{ kPa)} \qquad (11)$$

where ΔP (in kilopascals) is the difference between the tank and TVS pressures.

Equation (10) is useful for specifying the range of vent flow rates required to maintain constant pressure. Figure 6 shows that for a given pressure change rate, the required TVS flow rate decreases with an increasing value of the coefficient C.

In passive TVS operation, the conduction- and/or natural convection-dominated heat transfer process usually leads to a slow tank pressure response to the system operation. In reality, a significant amount of TVS flow will be vented before a reduction in tank pressure occurs. In addition, the tank fluid will be thermally stratified. Thus, the TVS flow rate and the total vented mass obtained from Eq. (10) are expected to be lower than the actual requirements.

Active Thermodynamic Vent System

Assuming that a constant-volume propellant tank is in a homogeneous state when the active TVS pressure-control method is applied, the homogeneous equation can be simplified and used to predict the rate of change in tank pressure:

$$\frac{dP}{dt} = \frac{\phi}{V}\left[Q_w + W_{mix} - m_{tvs}\Delta h_{tvs} - m_{tvs}h_{fg}\rho^*\right] \qquad (12)$$

where it has been assumed that saturated liquid ($x = 0$) is removed via a liquid acquisition device. It is noted that $W_{mix} - m_{tvs}\Delta h_{tvs}$ may be replaced by $-m_{mix}(h_{mix,o} - h_{mix,i})$.

To maintain constant tank pressure, the required energy removal rate by the TVS heat exchanger is

$$(m_{tvs}\Delta h_{tvs})_{dP/dt=0} = Q_w + W_{mix} - m_{tvs}h_{fg}\rho^* \qquad (13)$$

Thus, Eq. (12) can be expressed as

$$\frac{dP}{dt} = \frac{\phi}{V}Q_w\left[1 + \frac{W_{mix}}{Q_w} - \frac{m_{tvs}h_{fg}\rho^*}{Q_w}\right](1 - r_1) \qquad (14)$$

where the parameter r_1 is defined as

$$r_1 = \frac{m_{tvs}\Delta h_{tvs}}{(m_{tvs}\Delta h_{tvs})_{dP/dt=0}} \qquad (15)$$

The parameter r_1 represents the ratio of total heat actually removed from the mixer fluid to total heat required to be removed in order to have $dP/dt = 0$. Several observations can be made regarding Eq. (14):

1) If $m_{tvs} = 0$ ($r_1 = 0$) and $W_{mix} > 0$, then only fluid mixing is performed and Eq. (14) will yield Eq. (7).
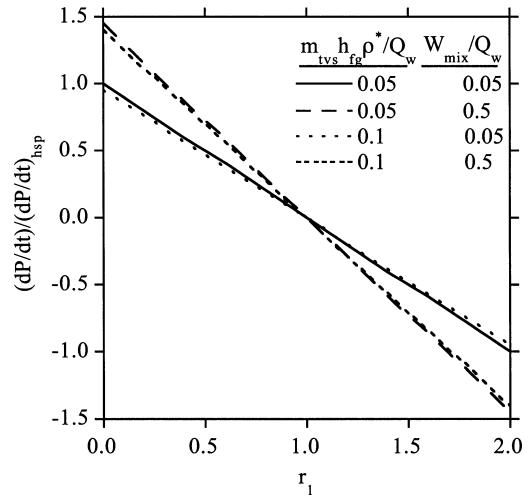
2) If $m_{tvs} > 0$ ($r_1 > 0$) and $W_{mix} = 0$, then passive TVS only is performed yielding Eq. (10).

3) If both $m_{tvs} > 0$ ($r_1 > 0$) and $W_{mix} > 0$, then active TVS is performed.

Equation (14) is functionally equivalent to

$$\frac{dP/dt}{\phi Q_w/V} = \frac{dP/dt}{(dP/dt)_{hsp}} = f\left[r_1, \frac{W_{mix}}{Q_w}, \frac{m_{tvs}h_{fg}\rho^*}{Q_w}\right] \qquad (16)$$

Based on Eq. (16), the performance of an active TVS can be evaluated as a function of relevant parameters. Figure 7 shows the effect of r_1 on the pressure change rate for $W_{mix}/Q_{mix} = 0.05$ and 0.5 and $m_{tvs}h_{fg}\rho^*/Q_w$ equal to 0.05 and 0.1, which are representative values for spacecraft tanks. If the actual heat removed by the TVS heat exchanger has a 50% variation from the required value for $dP/dt = 0$ (i.e., $r_1 = 0.5$ or 1.5), then $dP/dt = \pm 50$ and $\pm 75\%$ for $W_{mix}/Q_{mix} = 0.05$ and 0.5, respectively. The sensitivity of $(dP/dt)/(dP/dt)_{hsp}$ to r_1 increases as W_{mix}/Q_{mix} increases. Thus, lower jet flow rate and/or improvement of mixer efficiency, resulting in lower required mixer power, gives improved mixing performance.

The required energy removal rate specified by Eq. (13) to maintain $dP/dt = 0$ includes that required to offset heat addition from the mixer. A parameter r_2 representing the fraction of the total TVS mass flow rate used to remove this mixer-generated heat can be defined as

$$r_2 = \left[\frac{W_{mix}}{Q_w + W_{mix} - m_{tvs}h_{fg}\rho^*}\right]_{dP/dt=0} \qquad (17)$$

Figure 8 shows the effect of the mixer-power-to-volumetric-heating ratio W_{mix}/Q_w on the parameter r_2. It is observed that the effect of the ratio $m_{tvs}h_{fg}\rho^*/Q_w$ on r_2 is quite small for a wide range of mixer power inputs. However, for a given TVS flow rate, r_2 increases significantly with increasing W_{mix}/Q_w. For example, $W_{mix}/Q_w = 0.05$ has a value of r_2 equal to 0.04, whereas the value of r_2 for $W_{mix}/Q_w = 0.5$ is about 0.3. The design of an active TVS should have a value of W_{mix}/Q_w as low as possible.

Combining Eqs. (9) and (12) gives

$$\frac{dP}{dt} = \frac{\phi Q_w}{V}\left[1 + \frac{W_{mix}}{Q_w} - \frac{m_{tvs}h_{fg}}{Q_w}(C + \rho^*)\right]$$

$$= \left(\frac{dP}{dt}\right)_{hsp}\left[1 + \frac{W_{mix}}{Q_w} - \frac{m_{tvs}h_{fg}}{Q_w}(C + \rho^*)\right] \qquad (18)$$

Comparison of Eqs. (10) and (18) reveals that more TVS mass flow is vented by an active TVS than by a passive TVS because of the additional heat input by mixer power if C is equal for both systems. However, in the active TVS operation, the heat transfer process is a result of forced convection, which has a much faster response than the conduction/natural convection processes in the
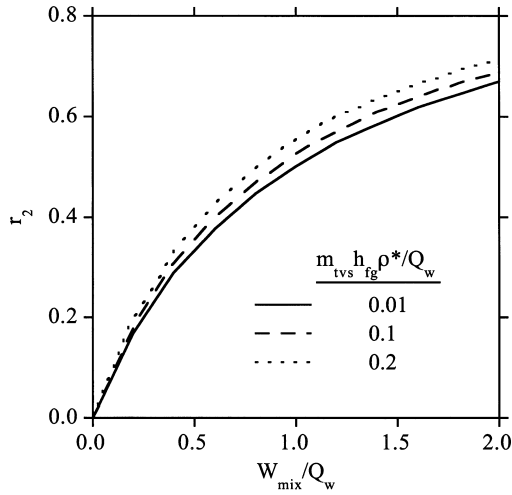
Fig. 8   Effect of mixer power on the parameter r_2.

passive TVS. Also, tank thermal stratification is largely destroyed in active TVS but not in passive TVS. Therefore, in reality, the homogeneous model is more appropriate for an active TVS than for a passive TVS. If the mixer power is not exceptionally large, the active TVS should be more efficient than a passive TVS. It is noted that in the design of an active TVS pressure-control experiment, one can evaluate the order of magnitude for each term on the right-hand side of Eq. (18).

Conclusions

An analysis of the self-pressurization of a cryogenic storage system and the performance of pressure-control devices such as fluid mixing, passive TVS, and active TVS has been presented. Simplified equations based on homogeneous models are given to predict the pressure-change rate of a self-pressurizing tank that has a known total heating rate with or without a mixer or TVS. It is assumed that the tank fluid is isothermal and at the saturation temperature corresponding to the tank pressure. The equations do not apply to cases where a noncondensable gas is present or if methods other than mixing or TVS are used to control tank pressure. Since the equations are developed from thermodynamic analysis only, factors such as surface heating uniformity, tank shape, wetted tank wall area, and liquid position are not considered, although they may have relevance to various practical applications. The various pressure control technologies are intended for use in low gravity, where the liquid is not settled and preferential venting of vapor will not be practical. Several conclusions are drawn from the analysis:

1) The pressure change rate of a tank increases with tank volumetric heating rate Q_w/V.

2) Fluid mixing reduces pressure rise rate by destroying thermal stratification. This is an appropriate method for moderate extension of short-term storage duration. An optimal design of the mixer device is one that has the lowest value of the product of mixer power and mixing time, $W_{mix}t_{mix}$, for a given tank pressure reduction rate. Intermittent mixing is preferred over continuous mixing. The mixing time and mixing power of a given system should be accurately predicted to obtain an efficient pressure control procedure (i.e., to minimize $W_{mix}t_{mix}$.)

3) For a given pressure change rate in a passive or active TVS, the required flow rate decreases with increasing $C(= \Delta h_{tvs}/h_{fg})$. An active TVS design should have a value of W_{mix}/Q_w as low as possible so that the fraction of total TVS flow rate required to remove the heat generated by the mixer is minimized. The determination of actual TVS heat exchanger and mixing device effectiveness is needed to predict the values of $m_{tvs}\Delta h_{tvs}$ and $m_{mix}\Delta h_{mix}$, respectively, required for either maintaining constant tank pressure ($dP/dt = 0$) or reducing the tank pressure at a desired rate.

Appendix: Numerical Approximation of the Energy Derivative

The energy derivative ϕ can be expressed in terms of average density ρ or specific volume v:

$$\phi = 1 \Big/ \rho \left(\frac{\partial u}{\partial P} \right)_\rho = v \Big/ \left(\frac{\partial u}{\partial P} \right)_v \qquad \text{(A1)}$$

The tank fill fraction β, fluid quality x, and density or specific volume ratio are related as follows:

$$(1 - x)/x = (\rho_f/\rho_g)[\beta/(1 - \beta)] = (v_g/v_f)[\beta/(1 - \beta)] \quad \text{(A2)}$$

where $\beta = $ percent fill/100. Rearranging gives an expression for x:

$$x = \{1 + (\rho_f/\rho_g)[\beta/(1 - \beta)]\}^{-1} = \{1 + (v_g/v_f)[\beta/(1 - \beta)]\}^{-1}$$
$$\text{(A3)}$$

The quality is used to calculate ρ or v, used in Eq. (A1):

$$\rho = [x/\rho_g + (1 - x)/\rho_f]^{-1} = \text{ constant} \qquad \text{(A4)}$$

$$v = xv_g + (1 - x)v_f = \text{constant} \qquad \text{(A5)}$$

The partial derivative in Eq. (1) is numerically approximated as

$$\left(\frac{\partial u}{\partial P} \right)_\rho = \left(\frac{\partial u}{\partial P} \right)_v \approx \frac{u_2 - u_1}{P_2 - P_1} \qquad \text{(A6)}$$

States 1 and 2 must have the same density (or specific volume). Therefore the qualities at these states must be calculated in order to calculate the respective internal energies u:

$$u = xu_g + (1 - x)u_f \qquad \text{(A7)}$$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{1/\rho - 1/\rho_f}{1/\rho_g - 1/\rho_f} \qquad \text{(A8)}$$

where ρ or v have been calculated using Eq. (A4) or (A5).

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